Mathematics Department
Inverclyde Academy

A Curriculum for Excellence

Strategies and Methodologies

Numeracy Outcomes and Experiences
3rd and 4th Level

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Subtraction by Decomposition

The explanation below attempts to illustrate the concept of subtraction by the method of decomposition.

Experiences and Outcomes: MNU2-03a, MNU3-03b, MNU 4-03a

```
  2  9* 3
- 1  7  8
```

* storeroom with 9 boxes

We have the problem of '3 – 8' here. Imagine there are only 3 bags of crisps on the shelf, but the customer wants 8 bags. '3 – 8' doesn't make sense here, since we can't have '– 5' bags of crisps.
So I go to the storeroom where I have 9 boxes of crisps, 10 bags in each box. I open 1 box and place the 10 bags on the shelf beside the 3 already there. Now there are 13 bags available, but only 8 boxes left in the storeroom.

So the sum is

$$\begin{array}{r} 2\ {}^{8}\cancel{9}\ {}^{1}3 \\ -\ 1\ 7\ 8 \\ \hline 5 \end{array}$$

Completing it: -

$$\begin{array}{r} 2\ {}^{8}\cancel{9}\ {}^{1}3 \\ -\ 1\ 7\ 8 \\ \hline 1\ 1\ 5 \end{array}$$

This type of argument can be extended to larger subtractions,

```
  3 9 7 4
- 2 5 8 7
```

```
  8 6 2 5
- 5 7 4 6
```

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Fractions of a quantity

Finding a fraction of a quantity builds on their knowledge from their work on fractions, decimals and percentages where they will learn what a fraction is and what it looks like.

Usually finding a fraction of a quantity is done mentally or without a calculator with some written working.

The preferred method for finding a fraction of a quantity is first to divide by the **denominator** (number on the bottom of the fraction) and then multiply that answer by the **numerator** (number on the top of the fraction).

Experiences and Outcomes: MNU2-07a, MNU3-07a, MNU4-07a

Example of Calculation	Suggested Strategy	Helpful Hints
Calculate mentally $\frac{1}{4}$ of 24.	Divide by the denominator	Mentally: • How many 'fours' are in 24? • 4 times what is 24? • Half it, half it again
Calculate $\frac{3}{5}$ of 60	Divide by the denominator, then multiply the answer by the numerator. $5\overline{)6^{1}0}$ $\quad 12$ then $\begin{array}{r} 12 \\ \times\ 3 \\ \hline \mathbf{36} \end{array}$	Can think about it as calculating $\frac{1}{5}$ (one fifth) first, then you can calculate two fifths and then three fifths, as required.

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Order of Operations – *BOMDAS*

"Operations" means sums like add, subtract, multiply, divide, squaring… If it isn't a number it is probably an operation.

But, when you see something like...

$$7 + (6 \times 5^2 + 3)$$

... which part should you calculate first?

Start at the left and go to the right? Or go from right to left?
Calculate them in the wrong order, and you will get a wrong answer!

Experiences and Outcomes: MTH2-03c, MTH4-03b

So, long ago people agreed always to follow certain rules when doing calculations. The rules are:

Order of Operations

Do sums in Brackets First. Example:

✓ $6 \times (5 + 3) = 6 \times 8 = \mathbf{48}$
✗ $6 \times (5 + 3) = 30 + 3 = 33$ (wrong)

Exponents (Powers, Roots) before Multiply, Divide, Add or Subtract. Example:

✓ $5 \times 2^2 = 5 \times 4 = \mathbf{20}$
✗ $5 \times 2^2 = 10^2 = 100$ (wrong)

Multiply or Divide before you Add or Subtract. Example:

✓ $2 + 5 \times 3 = 2 + 15 = \mathbf{17}$
✗ $2 + 5 \times 3 = 7 \times 3 = 21$ (wrong)

Otherwise just go left to right. Example:

✓ $30 \div 5 \times 3 = 6 \times 3 = \mathbf{18}$
✗ $30 \div 5 \times 3 = 30 \div 15 = 2$ (wrong)

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Order of Operations – *BOMDAS*

How Do I Remember? BOMDAS !

B	**B**rackets first
O	**Of** (ie Powers and Square Roots, etc.)
MD	**M**ultiplication and **D**ivision (left-to-right)
AS	**A**ddition and **S**ubtraction (left-to-right)

Multiply and Divide rank equally (and go left to right).

Add and Subtract rank equally (and go left to right)

Example: How do you work out **3 + 6 × 2** ?

Multiplication before **A**ddition:

First 6 × 2 = 12, then 3 + 12 = 15

Example: How do you work out **(3 + 6) × 2** ?

Brackets first:

First **(3 + 6) = 9**, then **9 × 2 = 18**

Example: How do you work out **12 / 6 × 3** ?

Multiplication and **D**ivision rank equally, so just go left to right:

First **12 / 6 = 2**, then **2 × 3 = 6**

Oh, yes, and what about 7 + (6 × 5^2 + 3) ?

7 + (6 × 5^2 + 3)	
7 + (6 × 25 + 3)	Start inside Brackets, and then use "Of" First
7 + (150 + 3)	Then Multiply
7 + (153)	Then Add
7 + 153	Brackets completed, last operation is add
160	DONE !

Measurement

The measurement of length and angles in maths is an essential practical skill which is necessary beyond the classroom. We would expect pupils to know how to position protractors and rulers/metre sticks/tape measures to measure, and how to obtain measurements with reasonable accuracy.
We would also expect pupils to be able to estimate lengths and angles, to choose the appropriate units that should be used, and to convert between commonly used units of measurements.

Experiences and Outcomes: MNU2-11a,b, MNU3-11a,b MNU4-11a

Accepted and preferred strategies and methodologies

- Know the commonly used units for length, area and volume such as millimetres, centimetres, metres, kilometres, square centimetres, square metres, cubic centimetres, cubic metres, millilitres and litres.
- Be able to choose the appropriate units for measurement.
- Be able estimate a length, area or volume, e.g. length of a teaspoon is 11cm, length of bedroom is 3m or volume of glass of juice is 250ml.
- Encourage pupils to round measurements suitably – we can only measure objects to a certain level of accuracy - my answer of 59^{o} is just as acceptable as your answer of 60^{o}.
- Pupils should be encouraged to estimate lengths or angles before measuring them.
- Be able to measure lengths in cm or mm and be able to convert between the two units.
- Using the skills of multiplying and dividing numbers by 10, 100 and 1000 be able to change between units of length, volume and weight.

mm	÷ 10 → ← x 10	**cm**	÷ 100 → ← x 100	**m**	÷ 1000 → ← x 1000	**km**

ml (= cm^3)	÷ 1000 → ← x 1000	**litre**

mg	÷ 1000 → ← x 1000	**g**	÷ 1000 → ← x 1000	**kg**

- Point out imperial units for length, referring to DIY tasks undertaken by parents/grandparents/etc. and imperial lengths still referred to in televised sports like golf, American Football, rugby or football.

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Conversion of Fractions - Decimals – Percentages

The equivalence of Fractions, Decimal fractions and Percentages is a vital part of numeracy work along with being able to calculate a fraction or a percentage of a quantity.

Often, if faced with a problem, it can be eased if an equivalent form of the fraction, decimal fraction or percentage is used. As such it is a useful skill if a pupil can readily change among the three forms if and when necessary.

There are four we would expect pupils to 'know' off hand and the others can be calculated when needed. When doing such calculations it is advisable to go via percentages to ease the calculation.

Experiences and Outcomes: MNU2-07b, MNU3-07a, MNU4-07a

Standard Equivalences to be learned

- $^1/_2$ = 0.5 = 50%
- $^1/_4$ = 0.25 = 25%
- $^3/_4$ = 0.75 = 75%
- $^1/_{10}$ = 0.1 = 10%

Decimals to Percentages and vice versa

Divide the percentage by 100 to get decimal: 32% = 0.32, 9% = 0.09, 17.5% = 0.175
(Pupils spot 'rule')

And multiply decimal by 100 to get percentage: 0.72 = 72%, 0.02 = 2%, 0.111 = 11.1%

Percentages to Fractions

Percentage goes over 100 and fraction is simplified down - $\frac{65}{100} = \overset{\div 5}{\frac{\cancel{65}}{\cancel{100}}} = \frac{13}{20}$ $\div 5$

Fractions to Percentages

Multiply top and bottom of the fraction to get 100 on the bottom line and then top line is the %

$\frac{12}{25} = \overset{\times 4}{\frac{12}{25}} = \frac{48}{100} = 48\%$ $\times 4$

Decimals to Fractions

Decimal changes to percentage then follow the above method $0.8 = 80\% = \frac{80}{100} = \overset{\div 20}{\frac{\cancel{80}}{\cancel{100}}} = \frac{4}{5}$ $\div 20$

Fractions to Decimals

Change fraction to percentage as above then percentage to decimal $\overset{\times 10}{\frac{7}{10}} = \frac{70}{100} = 70\% = 0.7$ $\times 10$

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Using Percentages – without a calculator

This is an important numeracy skill for everyday life. Percentages are used in many situations such as: shopping, VAT, to compare amounts i.e. test scores or simply as a means of displaying or calculating information.
As with a number of mathematical concepts the need for a good understanding of basic number facts is an essential skill.

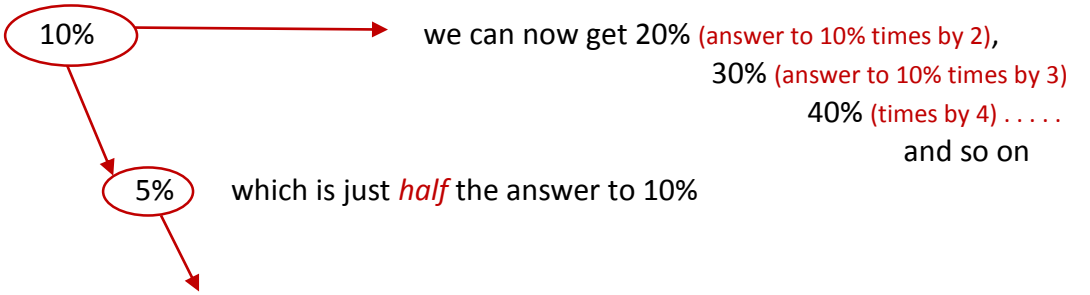
Experiences and Outcomes: MNU 2-07b, MNU 3-07a, MNU 4-07a

As mentioned on previous page, there are standard percentages that would make it easier to calculate many commonly occurring percentages in everyday life such as sales in a shop. These should be learned and they can help work out calculations mentally:

Standard Equivalences to be learned

- $^1/_2$ = 0.5 = 50% 'half the amount'
- $^1/_4$ = 0.25 = 25% 'half and half again'
- $^3/_4$ = 0.75 = 75% 'half and half again then add both answers'
- $^1/_{10}$ = 0.1 = 10% 'divide by ten'

10% is a very 'handy' percentage calculation to be able to do. Once we have 10%, by dividing by 10, we can use it go get lots of other commonly occurring percentages.



Examples

25% of 84 = **21**

Method

half 84 = 42
half again = 21

75% of £64.40 = **£48.30**

Method

half £64.40 = £32.20
half again = £16.10
now add = £32.20 + £16.10
= £48.30

30% of £26 = **£7.80**

Method

find 10% = £26 ÷ 10 = £2.60
now to get 30% times by 3
£2.60 x 3 = £7.80

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Using Percentages – without a calculator

Here is an illustration of calculating 15% and also working out a quantity as a percentage.

Percentage of an amount — Find 15 % of £180

Method 1 — Find 10% FIRST (divide by 10) and the find the correct multiple of this thereafter.

For 10% £180 ÷ 10 = £18

From there find 5% (half of 10% value)

For 5% £18 ÷ 2 = £9

For 15% (find total) £18 + £ 9 = **£27**

Method 2 — Find 15% of £180 FIRST (divide by 100) and the find the correct multiple of this thereafter.

For 1% £180 ÷ 100 =£1.80

For 15% 15 x £1.80 = **£27**

Finding a quantity as a percentage – 3rd Level only

In this type of question the numbers used will always be simple to work with.

Bike bought for £200 and sold for £ 160. Find the loss as a percentage of the purchase price.

Loss = £200 - £160 = £40

% Loss = $\frac{40}{200}$ x 100 cancel down in fraction

(This makes sum easier) % Loss = $\frac{1}{5}$ x 100 = **20%**

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Using Percentages – with a calculator

This is an important numeracy skill for everyday life. Percentages are used in many situations such as: shopping, VAT, to compare amounts i.e. test scores or simply as a means of displaying or calculating information.

Pupils are encouraged to learn how to use calculators from S2 onwards. The Department use the Casio FX83 range for Higher and National courses.

Experiences and Outcomes: MNU3-07a, MNU4-07a

Percentage of an amount

Find 15 % of £180
Method 1
Enter: 0.15 x 180
Press: =
= £27

Method 2
Enter: 15 ÷ 100 x 180
Press: Ans
= £27

Find 72% of £13
Method 1
Enter: 0.72 x 13
Press: =
= £9.36

Method 2
Enter: 72 ÷ 100 x 13
Press: =
= £9.36

If we continue to encourage the skill of converting percentages to decimal fractions then we can work out percentage increases decreases quickly in one sum on a calculator. The skill of using decimal fractions for percentages helps when using a spreadsheet to calculate percentages and this skill is used in ICT department in Inverclyde Academy.

Percentage increase/decrease calculations

15% off Sale
How much would you pay for a £19.99 shirt?

Method
100% — 15% off
Leaves 85%

85% of £19.99
= 0.85 x 19.99
= £16.99

Joan earns a salary of £16,850 a year.
She is told she is getting a 2.3% pay increase.
What is Joan's new salary?

Method
100% — 2.3% added on
so now have 102.3%

102.3% of £16 850
= 1.023 x 16850
= £17 237.55

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Graphs and Charts: Techniques and Language used

Graphs and Charts are important, not only in Maths, but in everyday life. They are used in almost every other curricular area in the school.

The following are teaching points and terminology used in Maths when discussing graphs.

Experiences and Outcomes: MNU2-20b, MNU3-20a, MTH2-21a, 3-21a, MNU4-20a, MNU 4-21a

Titles and Labels

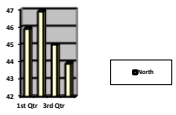
A graph of any type should have an overall title and each axis should be labelled to indicate what the numbers are representing

Scales

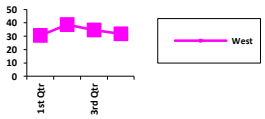
Any numeric scale should be evenly spaced and increase in equal steps, e.g. 5, 10, 15, . . One of the main teaching points is for a student to find out what each interval (or box on the paper) represents. It is important to count the gaps between two numeric markers and divide that increase by the number of gaps.

Types of Graphs

Bar graphs – the coloured bars height is the figure we read off.
Good for discrete data or opinions

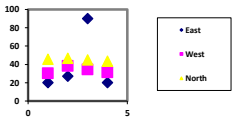


Line graphs – a series of points joined up by straight lines.
Good for continuous data and displaying trends.



Scatter graphs – Two quantities plotted against each other

- Positive correlation: as one increases the other increases
- Negative correlation: as one increases the other decreases
- No correlation: no relationship evident



Terminology

Line of Best fit is a line that can be drawn on a scatter graph to help estimate one variable given the other. This is done by eye, passing through as many points as possible with roughly the same number of points above and below the line.

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Mathematics Department Word Bank

Decimal places – number of digits after the decimal point

Difference – subtraction

Sum – addition

Product – answer to a multiplication sum

Quotient – answer to a division sum

Remainder – what is left over when a division sum is not exact

Multiple – a number generated when the given number is multiplied by another number
(**M**ultiples are numbers you **M**ake)

Factor – two numbers that multiply to make the given number
(**F**actors are numbers you **F**ind *inside* a number)

Integer – whole numbers including zero and negative whole numbers

Power – or the index, how many times a number is multiplying itself

Scientific notation/standard form – shorter way of writing really large or really small numbers

Square root – looking for a smaller number that times itself makes the given number

Numerator – top line of a fraction

Denominator – bottom line of a fraction, number doing the dividing.