1 Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots. **4**

(a) Express $2x^2 + 4x - 3$ in the form $a(x+b)^2 + c$.

(b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$.

2 Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

3 A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(a) Show that $(x-3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

(c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. **5**

4 Show that $x = -1$ is a solution of the cubic equation

$$x^3 + px^2 + px + 1 = 0$$ **7**

Hence find the range of values of p for which all the roots of the cubic equation are real. **1**

5 $f(x) = x^3 - x^2 - 5x - 3$.

(a) (i) Show that $(x+1)$ is a factor of $f(x)$.

(ii) Hence or otherwise factorise $f(x)$ fully. **5**

(b) One of the turning points of the graph of $y = f(x)$ lies on the x-axis.

Write down the coordinates of this turning point. **1**

6 (a) Write $x^2 - 10x + 27$ in the form $(x+b)^2 + c$. **2**

(b) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. **4**

7 Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p. **4**

8 (a) Write $f(x) = x^2 + 6x + 11$ in the form $(x+a)^2 + b$. **2**

(b) Hence or otherwise sketch the graph of $y = f(x)$. **2**

9 Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

10 $f(x) = 6x^3 - 5x^2 - 17x + 6$.

(a) Show that $(x - 2)$ is a factor of $f(x)$.

(b) Express $f(x)$ in its fully factorised form. **4**

11 A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.
What is the remainder when $f(x)$ is divided by $(x - 1)$?

A 0
B 2
C 3
D 4

12 Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(a) Find $g(f(x))$. **2**

(b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

(d) Hence solve $g(f(x)) + xh(x) = 0$. **1**

13 (a) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$. **6**

14 If $x^2 - 6x + 14$ is written in the form $(x - p)^2 + q$, what is the value of q?

A −22
B 5
C 14
D 50

15 (*a*) (i) Show that $(x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully. **5**

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. **1**

(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G. **5**

(*d*) This tangent meets the curve again at the point H.

Write down the coordinates of H. **1**

16 What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x-2)$?

A 0

B 3

C 4

D 8

Marks

17 (*a*) Given that $(x-1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. **4**

(*b*) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x-coordinate and determine the nature of this point. **5**

18 For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(*a*) Determine the values of a and b. **4**

(*b*) Hence factorise the polynomial completely. **3**

19 (*a*) Show that $(x-1)$ is a factor of $x^3 - 6x^2 + 9x - 4$ and hence factorise $x^3 - 6x^2 + 9x - 4$ fully. **4**

20 $3x^2 + 12x + 17$ is expressed in the form $3(x+p)^2 + q$.

What is the value of q?

A 1

B 5

C 17

D –19

21 Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. **3**

22 Solve $1 - 2x - 3x^2 > 0$, where x is a real number.

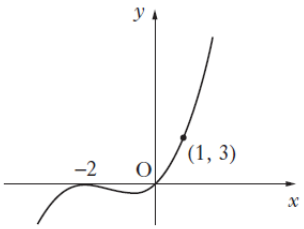
A $x < -1$ or $x > \frac{1}{3}$

B $-1 < x < \frac{1}{3}$

C $x < -\frac{1}{3}$ or $x > 1$

D $-\frac{1}{3} < x < 1$

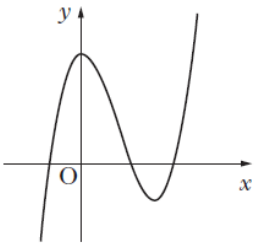
23 The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points $(-2, 0)$, $(0, 0)$ and $(1, 3)$.



What are the values of a and k?

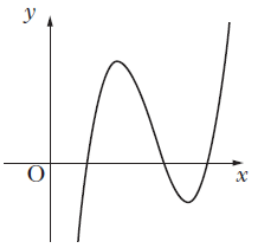
	a	k
A	−2	$\frac{1}{3}$
B	−2	3
C	2	$\frac{1}{3}$
D	2	3

24 The diagram shows a cubic curve with equation $y = f(x)$.

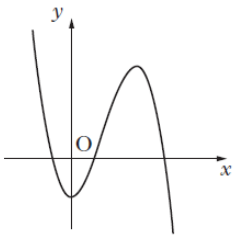


Which of the following diagrams could show the curve with equation $y = -f(x - k)$, $k > 0$?

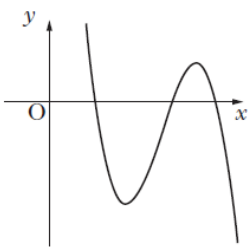
A



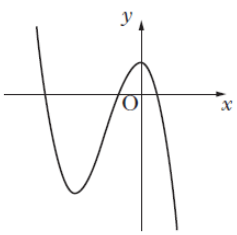
C



B



D



25 Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.

A -32

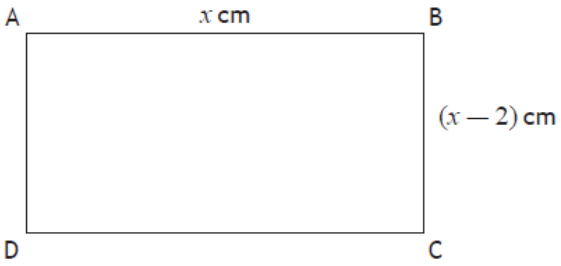
B -24

C 48

D 56

MARKS

26 ABCD is a rectangle with sides of lengths x centimetres and $(x - 2)$ centimetres, as shown.



If the area of ABCD is less than $15\,\text{cm}^2$, determine the range of possible values of x. 4

27 Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully. 4

28 Solve $6 - x - x^2 < 0$.

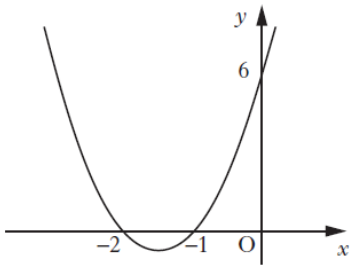
A $-3 < x < 2$

B $x < -3, x > 2$

C $-2 < x < 3$

D $x < -2, x > 3$

29 A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.



What is the equation of the parabola?

A $y = 6(x - 1)(x - 2)$

B $y = 6(x + 1)(x + 2)$

C $y = 3(x - 1)(x - 2)$

D $y = 3(x + 1)(x + 2)$

Solutions

1 $k < \frac{-1}{4}$

(a) $2(x+1)^2-5$ (b) (-1, -5) Min tp

2. k = 24

3(a) $(x - 3)(2x - 3)(x + 1)$

(b) (3, 0) (1.5, 0) (-1, 0) (0, 9)

(c) Max = 9, Min = -35

4 p < -1 p > 3

5(a) $(x + 1)(x + 1)(x - 3)$

(b) (-1, 0)

6(a) $(x - 5)^2 + 2$

(b) $g'(x) = (x - 5)^2 + 2 > 0$ therefore always increasing

7 $p^2+24>0$ for any value of p therefore roots are real

8(a) $(x+ 3)^2 + 2$

(b) Graph drawn with min tp at (-3, 2) and cuts y axis at 11.

9 $b^2 - 4ac = -11 < 0$ therefore doesn't intersect

10(a)

(b) $(x - 2)(3x - 1)(2x + 3)$

11 D

12(a) $3x^3-2$

(b) $3x^3 - 2 + x(4x - 5)$

(c) $(x - 1)(3x + 1)(x + 2)$

(d) x = 1, $x = \frac{-1}{3}$, x = -2

13(a) $(x - 4)(x - 2)(x + 1)$

X = 4, x = 2, x = -1

14 B

15(a) $(x - 1)(x - 1)(2x + 5)$

(b) $x = 1, x = \frac{-5}{2}$

(c) (1, -1)

(d) $\left(\frac{-5}{2}, -8\right)$

16 C

17(a) $(x - 1)(x^2 + 4x + 5)$ second bracket can't be factorised as discriminant < 0

(b) x = 1 minimum

18(a) a = -25 b = -26

(b) $(x + 1)(6x + 13)(x - 2)$

19 $(x - 1)(x - 1)(x - 4)$

20 B

21 $2(x + 3)^2 - 17$

22 B

23 C

24 B

25 B

26 $-3 < x < 5$

27 $(x + 3)(x - 4)(x - 2)$

28 C

29 D