1 A sequence is defined by the recurrence relation

$u_{n+1} = (k - 2)u_n + 5$ with $u_0 = 3$.

For what values of k does this sequence have a limit as $n \to \infty$?

A $-3 < k < -1$

B $-1 < k < 1$

C $1 < k < 3$

D $k < 3$

2 A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$.

What is the value of u_4?

A $2\frac{1}{9}$

B $2\frac{1}{3}$

C 3

D 30

3 A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}1u_n + 8$, with $u_1 = 11$.

Here are two statements about this sequence:

(1) $u_0 = 9{\cdot}1$;

(2) The sequence has a limit as $n \longrightarrow \infty$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

4 The first three terms of a sequence are 4, 7 and 16.

The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

Find the values of m and c. **4**

5 A sequence is defined by the recurrence relation $u_{n+1} = 3u_n + 4$, with $u_0 = 1$.

Find the value of u_2.

A 7

B 10

C 25

D 35

6 A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3} f_n + 32, \qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4} t_n + 13, \qquad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day. **1**

(b) Determine whether or not either of them will eventually escape from the well. **5**

7 For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(*a*) Why do these sequences have a limit? **2**

(*b*) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$.
Find the value(s) of x. **7**

8 (*a*) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.
Write down the values of u_1 and u_2. **1**

(*b*) A second sequence is given by 4, 5, 7, 11, ...
It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.
Find the values of p and q. **3**

(*c*) Either the sequence in (*a*) or the sequence in (*b*) has a limit.

(i) Calculate this limit.

(ii) Why does the other sequence not have a limit? **3**

9 A sequence is generated by the recurrence relation $u_{n+1} = \frac{1}{4}u_n + 7$, with $u_0 = -2$.
What is the limit of this sequence as $n \to \infty$?

10 A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$.
What is the value of u_2?

11 A sequence is defined by the recurrence relation $u_{n+1} = 0.8u_n + 12$, $u_0 = 4$.

(*a*) State why the recurrence relation has a limit.

(*b*) Find this limit.

12 (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$.
Find the value of k which produces a sequence with a limit of 4. **2**

(*b*) A sequence satisfies the recurrence relation

$$u_{n+1} = mu_n + 5,\ u_0 = 3$$

(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. **5**

13 A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$.

(a) Write down the condition on k for this sequence to have a limit.

(*b*) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k.

14 **A recurrence relation is defined by $u_{n+1} = pu_n + q$. where $-1 < p < 1$ and $u_0 = 12$.**

(*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q.

(*b*) Find the limit of this recurrence relation as $n \to \infty$.

Solutions

1. D

2. C

3. C

4. m = 3, c = -5

5. A

6.(a) 22.75ft (b) Toad escapes, frog will not

7(a) For sin x between that range the range of values is 0 and 1 (b) $x = \frac{3\pi}{2}$

8(a) U1 = 8, U2 = -4 (b) p = 2, q = -3 (c)(i) L = 0 (ii) No limit as 2>1

9 L = 9.33333 or $\frac{28}{3}$

10 U2 = 13

11(a) Limit exists as -1< 0.8 < 1 (b) 60

12(a) k = -0.25 (b)(i) u1 = 3m + 5 u2 = m(3m + 5) + 5 (ii) m = -2

13(a) -1 < k < 1 (b) k = 0.4

14(a) $p = \frac{1}{3}$ q = 11 (b) 16.5